Exercise 63

Prove the formula for $(d/dx)(\cos^{-1} x)$ by the same method as for $(d/dx)(\sin^{-1} x)$.

Solution

Let $y = \cos^{-1} x$. Then

$$\cos y = x. \tag{1}$$

Differentiate both sides with respect to x.

$$\frac{d}{dx}(\cos y) = \frac{d}{dx}(x)$$

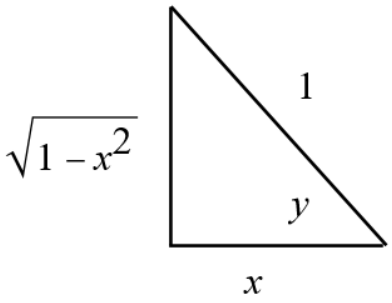
Use the chain rule to differentiate $y = y(x)$.

$$(-\sin y)\frac{dy}{dx} = 1$$

Solve for dy/dx.

$$\frac{dy}{dx} = -\frac{1}{\sin y}$$

Draw the implied right triangle from equation (1) in order to determine the sine of y. Use the Pythagorean theorem to determine the missing side.



$$\sin y = \frac{\sqrt{1-x^2}}{1} = \sqrt{1-x^2}$$

Therefore,

$$\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}.$$